Derivation of Analytical Dyadic Green s Function Modifications for Microstrip Attenuation in Transmission Layered Structures

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Abstract **Rigorous derivation of the correction to the dyadic Green s function for a microstrip structure containing complex layered media is done for imperfect metallization. A hierarchy of formulas is found consistent with a full-wave electromagnetic code employing zero thickness extent conductors for the guiding structure metal. At the bottom of the hierarchy are formulas which are only dependent on the conductor geometry and material properties. Numerical examples show the sensitivity of the attenuation constant to these formulas.**

I. Introduction

Spectral domain codes based upon the moment method which solve the electromagnetic integral equation problem for a multi-layered structure can place their prime focus on the material properties of the layers. Studies of the various anisotropies, nonreciprocities, crystallographic rotations, biasing field orientations in ferromagnetic or ferroelectric materials, can all be done with such codes [1], [2]. Theoretical investigations of this nature can be useful in deciding what structures to construct for integrated circuit applications [3]. Device behavior will most often be decided by the resulting phase properties of the transmission structures (phase shifting, coupling, delay, for example). However, being able to reasonably accurately predict the loss consequences of the imperfect metallizations used in actual devices may be necessary for the eventual development of working devices in circuits.

Full-wave electromagnetic codes employing zero thickness extent conductors for the guiding structure metal can be modified to account for the finite conductivity of the metallization and thickness. The most straight forward modifications, which are self-consistent in that they avoid any perturbational approaches, will alter the structure interfacial dyadic Green s function [4], where the guiding metal is located, with an expression which only depends upon the metal properties. These modifications can be employed most broadly for different spectral domain codes. Such modifications can be shown to come from a more general class of modifications which require additional structure dyadic Green s functions at the interface. Governing equations which employ these additional structure dyadic Green s functions at the interface have a form which is applicable to different spectral domain codes, but each code will require its own specific interfacial dyadic Green s functions to actually effect the modification. Thus the more general approach can not be as easily applied to different spectral domain codes.

For situations where the metallizations are very thick compared to other structure geometric dimensions, where the metallization thicknesses are expected to significantly alter the basic device behavior, or when it is desired to obtain the nearly exact field distribution in the metallization vicinity, the approach discussed in this paper should not be used. Other methods are available.

II. General Theory

At the guiding metal interface

$$\mathbf{E} = \mathbf{E}_d + \mathbf{E}_c \tag{1}$$

$\mathbf{E}$ is the total electric field vector at the interface. $\mathbf{E}_\mathrm{d}$ is the electric field vector at the interface where there is a dielectric mismatch. $\mathbf{E}_\mathrm{c}$ is the electric field vector at the interface where there is a conductor. $\mathbf{E}_\mathrm{d}$ and $\mathbf{E}_\mathrm{c}$ are zero outside their existence range. Since we will test this relationship with surface currents, it is the tangential form which is of greatest interest.

$$\mathbf{E}_{\tan} = \mathbf{E}_{d,\,\tan} + \mathbf{E}_{c,\,\tan} \tag{2}$$

Multiply the total electric field by a test surface current vector $\mathbf{J}_{\mathrm{sj}}$ and integrate over the interfacial coordinate x. The result is

$$\int_0^b \mathbf{E}_{\tan} \cdot \mathbf{J}_{sj}\, dx = \int_0^b \mathbf{E}_{d,\tan} \cdot \mathbf{J}_{\mathrm{sj}}\, dx + \int_0^b \mathbf{E}_{c,\tan} \cdot \mathbf{J}_{sj}\, dx \tag{3}$$

Because $\mathbf{E}_\mathrm{d}$ and $\mathbf{J}_{\mathrm{sj}}$ are complementary functions on the finite x-space, and using Parseval's theorem, we obtain

$$\sum_{n=-\infty}^{\infty} \left[\tilde{\mathbf{E}}_{\tan} - \left\langle \tilde{\mathbf{E}}_{c,\tan}(n,\ y) \right\rangle_{av} \right] \cdot \tilde{\mathbf{J}}_{sj}(n) = 0 \tag{4}$$

The term in wedge brackets represents some transform averaging over the thickness of the conductor, say over slices taken at different y values $y \in [0,\ t\ ]$. This averaging is not unique.

III. Surface Impedance and Dyadic Green s Function

Here will consider the conductor to be thick enough that it can be viewed as being composed of an upper piece of one material having a surface impedance Z_{su} and a lower piece having Z_{sl} . Electric field at either surface is given by

$$\tilde{\mathbf{E}}_s = Z_s\left(\hat{n} \times \tilde{\mathbf{H}}_s\right) \tag{5}$$

$\hat{n}$ is the surface normal pointing from the metal into the upper or lower region. We find that

$$\begin{aligned} \left\langle \tilde{\mathbf{E}}_{c,\tan}(n,y) \right\rangle_{av} &= \left\langle \tilde{\mathbf{E}}_{u,\tan}, \tilde{\mathbf{E}}_{l,\tan} \right\rangle_{av} \\ &= \sum_{i=1}^{n} \hat{n}_u \times \left\langle Z_{su}\tilde{\mathbf{H}}_{sui}, -Z_{sl}\tilde{\mathbf{H}}_{sli} \right\rangle_{av} a_i \end{aligned} \tag{6}$$

Linearity property of the averaging operator $\langle \; \rangle_{av}$ has been used to extract the coefficients a_i and the vector $\hat{n}_u = \hat{y}$. Upper and lower surface magnetic fields may be related by

$$\hat{n}_u \times \tilde{\mathbf{H}}_{sui} = \hat{n}_u \times \tilde{\mathbf{H}}_{sli} + \tilde{\mathbf{J}}_{si} \tag{7}$$

$$\tilde{\mathbf{H}}_{sli} = \mathbf{G}_{\mathbf{H}_S\mathbf{J}}\tilde{\mathbf{J}}_{si} \tag{8}$$

Relation (7) is exact at a single interface, so it must be viewed as an approximation for the finite thickness metal, somewhat convincingly since the fields in it are tied together by the single surface current characteristic of infinitely thin Green s function spectral domain solvers. Equation (8) may be used to eliminate unknown upper field $\tilde{\mathbf{H}}_{sui}$ by using a magnetic dyadic Green s function for the unknown lower field $\tilde{\mathbf{H}}_{sli}$, giving the modified

$$\mathbf{G}'_{\mathbf{E}_S\tilde{\mathbf{J}}_{si}} = \mathbf{G}_{\mathbf{E}_S\tilde{\mathbf{J}}_{si}} - \left\langle Z_{su}\left(\hat{n}_u \times \mathbf{G}_{\mathbf{E}_S\tilde{\mathbf{J}}_{si}} + 1\right), -Z_{sl}\left(\hat{n}_u \times \mathbf{G}_{\mathbf{E}_S\tilde{\mathbf{J}}_{si}}\right) \right\rangle_{av} \tag{9}$$

The above reduction is valid if the averaging operation is strictly linear, in actuality required in a rigorous sense since the Green's function approach is a linear process. Equation (9) is a rather remarkable result, as it allowed us to write down by inspection the new Green's function for the finite sized and finite conductivity metal strip in our anisotropic layered structure. We will not pursue this approach further, other than to say that although the form has been presented, it does not prove that in fact such a closed form representation can be found. Instead, in order to have some specific rules for constructing the average, resort to a procedure found in [5]. For any two spectral vectors $\tilde{\mathbf{A}}$ and $\tilde{\mathbf{B}}$, the average is

$$\left\langle \tilde{\mathbf{A}}, \tilde{\mathbf{B}} \right\rangle_{av} = \frac{A_x|A_x| + B_x|B_x|}{|A_x| + |B_x|}\hat{x} + \frac{A_z|A_z| + B_z|B_z|}{|A_z| + |B_z|}\hat{z} \tag{10}$$

Inversion of the magnitudes in formula (10) will make the averaging operator nonlinear, showing that such a construction to account for finite thickness and conductivity of the metal must be an approximation, since the actual problem is linear in the current $\tilde{\mathbf{J}}_s$. Nevertheless, we will enlist this formula to have something definite to discuss. From (10),

$$\begin{aligned} \left\langle \tilde{\mathbf{E}}_{c,\tan}(n,y) \right\rangle_{av,i} &= \frac{Z_{su}\tilde{J}_{suxi}\left|Z_{su}\tilde{J}_{suxi}\right| + Z_{sl}\tilde{J}_{slxi}\left|Z_{sl}\tilde{J}_{slxi}\right|}{\left|Z_{su}\tilde{J}_{suxi}\right| + \left|Z_{sl}\tilde{J}_{slxi}\right|}\hat{x} \\ &+ \frac{Z_{su}\tilde{J}_{suzi}\left|Z_{su}\tilde{J}_{suzi}\right| + Z_{sl}\tilde{J}_{slzi}\left|Z_{sl}\tilde{J}_{slzi}\right|}{\left|Z_{su}\tilde{J}_{suzi}\right| + \left|Z_{sl}\tilde{J}_{slzi}\right|}\hat{z} \end{aligned} \tag{11}$$

Equation (11) produces what can only be called an improper Green's function. Consider what happens when the upper and lower surface impedances are equal, $Z_{su} = Z_{sl}$,

$$\mathbf{G}'_{\mathbf{E}_S\mathbf{J}_{si}} = \mathbf{G}_{\mathbf{E}_S\mathbf{J}_{si}} - Z_s\begin{bmatrix} \dfrac{\tilde{J}_{suxi}\left|\tilde{J}_{suxi}\right| + \tilde{J}_{slxi}\left|\tilde{J}_{slxi}\right|}{\left(\left|\tilde{J}_{suxi}\right| + \left|\tilde{J}_{slxi}\right|\right)\tilde{J}_{sxi}} & 0 \\ 0 & \dfrac{\tilde{J}_{suzi}\left|\tilde{J}_{suzi}\right| + \tilde{J}_{slzi}\left|\tilde{J}_{slzi}\right|}{\left(\left|\tilde{J}_{suzi}\right| + \left|\tilde{J}_{slzi}\right|\right)\tilde{J}_{szi}} \end{bmatrix} \tag{12}$$

This type of Green's function only has meaning when used to reconstruct the $\tilde{\mathbf{E}}_{si}'$ difference field found in the testing procedure (4). And it is not unique - only when multiplied by $\tilde{\mathbf{J}}_{si}$ is the result unique. $\mathbf{G}'_{\mathbf{E}_S\tilde{\mathbf{J}}_{si}}$ has been shown to demonstrate the intrinsic complexity of trying to get a tractable modified Green's function using the averaging method in (10).

IV. Tangential Field Behavior in Strip

Detailed field behavior within the strip metal can be found by utilizing the Maxwell curl equations. If L, w >> δ for all guiding metals encountered, the strip will look like a large uniform sheet in the xz-plane. Thus $\partial/\partial x \to 0$ and $\partial/\partial z \to 0$ and for harmonic conditions where the time variation is $e^{j\omega t}$, these equations take the form

$$\frac{\partial E_z}{\partial y}\hat{x} - \frac{\partial E_x}{\partial y}\hat{z} = -j\omega\mu_m\mathbf{H} \; ; \; \frac{\partial H_z}{\partial y}\hat{x} - \frac{\partial H_x}{\partial y}\hat{z} = \left(j\omega\varepsilon_m + \sigma_m\right)\mathbf{E} \tag{13}$$

In order to learn what the tangential components do when traveling through the strip in the normal y-direction, we

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will exclude normal E_y or H_y field components, making the waves TEM_y. Then (13) becomes

$$\frac{\partial \mathbf{E}_{\tan}}{\partial y} - j\omega\mu_m \hat{y}\times\mathbf{H}_{\tan} = 0 \;;\; \frac{\partial \mathbf{H}_{\tan}}{\partial y} + \left(j\omega\varepsilon_m + \sigma_m\right)\hat{y}\times\mathbf{E}_{\tan} = 0 \tag{14}$$

Field solution within the metal strip is

$$\mathbf{E}_{\tan}(y) = \mathbf{E}_{sl}\cosh\gamma y + \eta_m \hat{y}\times\mathbf{H}_{sl}\sinh\gamma y$$

$$\mathbf{H}_{\tan}(y) = \mathbf{H}_{sl}\cosh\gamma y - \frac{1}{\eta_m}\hat{y}\times\mathbf{E}_{sl}\sinh\gamma y \tag{15}$$

Upper and lower tangential fields for the metal strip are, using (15),

$$\mathbf{E}_{su} = \hat{n}_u \times\left[Z_{uu}\mathbf{H}_{su} + Z_{ul}\mathbf{H}_{sl}\right] \;;\; \mathbf{E}_{sl} = \hat{n}_l \times\left[Z_{lu}\mathbf{H}_{su} + Z_{ll}\mathbf{H}_{sl}\right]$$

$$Z_{uu} = Z_{ll} = \eta_m \coth\gamma t \;;\; Z_{ul} = Z_{lu} = -\frac{\eta_m}{\sinh\gamma t}$$

$$\gamma_m = \sqrt{j\omega\mu_m\left(j\omega\varepsilon_m + \sigma_m\right)} \;;\; \eta_m = \frac{j\omega\mu_m}{\gamma_m} \tag{16}$$

For metals like copper, silver, and gold, it is indeed true that $\sigma_m >> j\omega\varepsilon_m$ [6] making $\gamma_m = (1+j)/\delta_m$, $\eta_m = (1+j)/\sigma\delta_m$, $\delta = 1/\sqrt{\pi f\mu_m\sigma_m} = 2.09/\sqrt{f}\sqrt{\sigma_{Cu}/\sigma_m}\;\mu m$ (f in GHz). Skin depth δ = 0.66, 0.21 µm at f = 10, 100 GHz in copper. Thus the assumption that L, w >> δ holds if w ≥ 5, 2 µm at f = 10, 100 GHz, because we are taking L≥ w.

V. Modified Dyadic Green s Function For Finite Metal Thickness

The expression for average conductor field is more complicated than (11) based upon (16), and we won t show it here. Instead, look at a limiting case when one can take $\tilde{\mathbf{J}}_{sui} \rightarrow 0$, not an unreasonable assignment for a strip configuration when the preponderance of the material is in a substrate form (In symmetric stripline this is an invalid assignment.)

$$\left\langle \tilde{\mathbf{E}}_{c,\tan}(n,y)\right\rangle_{av,i} = \frac{Z_{ll}\left|Z_{ll}\right| - Z_{ul}\left|Z_{ul}\right|}{\left|Z_{ll}\right| + \left|Z_{ul}\right|}\tilde{\mathbf{J}}_{si} \tag{17}$$

Expression (17) contains both diagonal and non-diagonal surface impedance elements, apparent if we write (16) as

$$\begin{bmatrix}\mathbf{E}_{su}\\ \mathbf{E}_{sl}\end{bmatrix} = \begin{bmatrix}\hat{n}_u & \hat{n}_l\end{bmatrix}\times\begin{bmatrix}Z_{uu} & Z_{ul}\\ Z_{lu} & Z_{ll}\end{bmatrix}\begin{bmatrix}\mathbf{H}_{su}\\ \mathbf{H}_{sl}\end{bmatrix} \tag{18}$$

Using the impedance element definitions in (16), the coefficient in (17) can be evaluated to be

$$C_Z = \frac{1}{\sigma_m t}\frac{(1+j)t/\delta}{\tanh\left[(1+j)t/\delta\right]}\times \left\{\frac{1+\dfrac{1}{\cosh\left[(1+j)t/\delta\right]}\dfrac{1}{\left|\cosh\left[(1+j)t/\delta\right]\right|}}{1+\dfrac{1}{\left|\cosh\left[(1+j)t/\delta\right]\right|}}\right\} \tag{19}$$

Evaluating the averaging operator as a simple sum (not a weight), (17) is replaced by

$$\left\langle \tilde{\mathbf{E}}_{c,\tan}(n,y)\right\rangle_{av,i} = \left(Z_{ll} - Z_{ul}\right)\tilde{\mathbf{J}}_{si} \tag{20}$$

This gives

$$C_Z = \frac{1}{\sigma_m t}\frac{\left(1+j\right)t/\delta}{\tanh\left[\left(1+j\right)t/\delta\right]}\left\{1+\frac{1}{\cosh\left[\left(1+j\right)t/\delta\right]}\right\} \tag{21}$$

Finally, if we neglect the non-diagonal matrix element Z_{ul} in the field impedance (16),

$$C_Z = \frac{1}{\sigma_m t}\frac{\left(1+j\right)t/\delta}{\tanh\left[\left(1+j\right)t/\delta\right]} \tag{22}$$

The three cases in (19), (21), and (22) generate the three modified dyadic Green s functions. They are respectively,

$$\mathbf{G}_{\mathbf{E}_s\mathbf{J}_{si}}{}' = \begin{bmatrix} G_{\mathbf{E}_s\mathbf{J}_{si}xx} - C_Z & G_{\mathbf{E}_s\mathbf{J}_{si}xz}\\ G_{\mathbf{E}_s\mathbf{J}_{si}zx} & G_{\mathbf{E}_s\mathbf{J}_{si}zz} - C_Z\end{bmatrix}$$

$$\mathbf{G}_{\mathbf{E}_s\mathbf{J}_{si}}{}' = \begin{bmatrix} G_{\mathbf{E}_s\mathbf{J}_{si}xx} - \left(Z_{ll} - Z_{ul}\right) & G_{\mathbf{E}_s\mathbf{J}_{si}xz}\\ G_{\mathbf{E}_s\mathbf{J}_{si}zx} & G_{\mathbf{E}_s\mathbf{J}_{si}zz} - \left(Z_{ll} - Z_{ul}\right)\end{bmatrix}$$

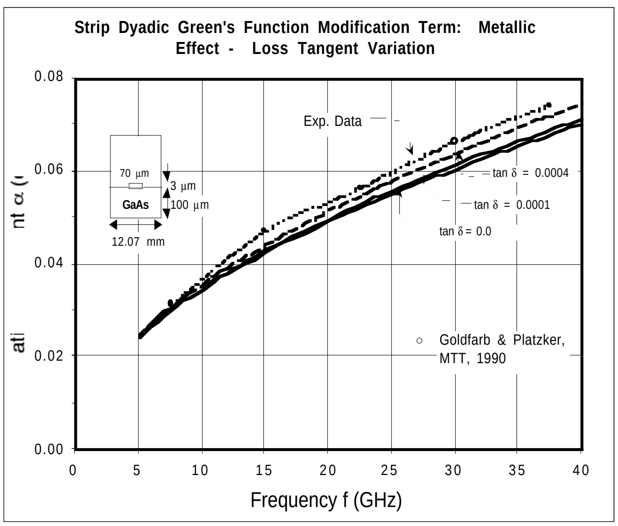
$$\mathbf{G}_{\mathbf{E}_s\mathbf{J}_{si}}{}' = \begin{bmatrix} G_{\mathbf{E}_s\mathbf{J}_{si}xx} - Z_{ll} & G_{\mathbf{E}_s\mathbf{J}_{si}xz}\\ G_{\mathbf{E}_s\mathbf{J}_{si}zx} & G_{\mathbf{E}_s\mathbf{J}_{si}zz} - Z_{ll}\end{bmatrix} \tag{23}$$

There are three regimes of metal thickness: $t >> \delta$, t ¯ δ, $t << \delta$. When $t >> \delta$, $C_Z = 1/\sigma_m\delta$. In the other extreme when $t << \delta$, $C_Z = 1/\sigma_m t \times (1,2,1)$. It is clear from this last result that formulas (19) and (22) may be the preferable ones to employ because of the unity limit expected for $t/\delta \rightarrow 0$. Equation (21) may be repairable by dividing it by a factor of 2.

VI. Numerical Results for the Analytical Limiting Modified Dyadic Green s Function Formulas

It is possible to validate the some of the concepts here in a non-anisotropic system. We choose microstrip over a GaAs substrate which was measured for various geometric dimensions over the nominal 5 - 40 GHz frequency range [7]. To compare our theoretical dyadic Green s function modification work to experiment, we must also include dielectric loss to obtain the total loss. The code (basis function number $n_x = n_z = 3$ and spectral number n = 200) was compared to [8] for microstrip, which is also a full—wave calculation. Two loss tangent cases were investigated, $\tan\delta = 2.0 \times 10^{-4}$ and 1.0×10^{-1} - agreement with our code results is excellent, within 1 %.

Now return to the experimental GaAs microstrip case with w = 70 µm wide microstrip, with substrate height h_s = 100 µm, relative dielectric constant ε_r = 12.9, box height above substrate (air region) = h_a = 10 mm, and box width = b = 12.07 mm. This b choice also is consistent with [9]. The figure plots the experimental and theoretical [using (22)] results for varying loss tangent ($n_x = n_z$ = 4 and n = 300), with metal thickness t = 3 µm and conductivity $\sigma_s = 4.1 \times 10^7$ S/m (mostly gold). Agreement is within a few percent. Formula (19) produces α slightly below (22), being 12, 4.4, 0.88, 0.26, 0.096 % at 5, 10, 20, 30, 40 GHz ($\tan\delta$ = 0). Formula (21) produces α slightly below or above (22), being —5.9, -6.2, 0.14, 0.34, 0.12 % at 5, 10, 20, 30, 40 GHz ($\tan\delta$ = 0).



VII. Conclusion

This paper has shown how to systematically develop the formalism for finding dyadic Green s function modifications, including obtaining analytical formulas, for zero thickness conductor spectral domain codes when studying loss of microstrip configurations. Approach presented here has been followed also for anisotropic ferroelectric coplanar structures, allowing description of experimental attenuation results [10], [11].

Acknowledgement

The author wishes to acknowledge the software assistance of Maurice Daniel of DCS Corp. and the support of DARPA under the FAME (Frequency Agile Materials for Electronics) program.

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